Estimating image blur in the wavelet domain

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Abstract— In this paper, a wavelet based method is proposed to estimate the blur in an image using information contained in the image itself. We look at the sharpness of the sharpest edges in the blurred image, which contain information about the blurring. Specifically, a smoothness measure, the Lipschitz exponent, is computed for these sharpest edges. A relation between the variance of a gaussian point spread function and the magnitude of the Lipschitz exponent is shown, which is only dependent on the blur in the image and not on the image contents. This allows us to estimate the variance of the blur directly from the image itself.

I. Introduction

Blurring of edges in an image occurs in many different fields. Image blur is modelled as:

$$g(x,y) = (h * f)(x,y) + n(x,y) \tag{1}$$

with $g(x,y)$ the blurred image, $f(x,y)$ the unknown sharp image and $h(x,y)$ the point spread function (PSF). The symbol $*$ represents the convolution operator, and models the image blur. It is in fact the response of the imaging system to an ideal point source. The term $n(x,y)$ represents additive image noise.

This image degradation obscures part of the information present in the image. The goal of image restoration, is to recover this information as good as possible, and it is applied in astronomy, medical imaging, microscopy, ...). Sometimes, one has information about the image blur, but not always. In the case no information about the blur is available, one has to estimate the blur to restore the ideal image $f(x,y)$ from degraded data $g(x,y)$.

In this paper, a method is proposed to estimate the PSF in an image by looking how sharp the sharpest edges in a blurred image still are, in order to find information about the PSF. It estimates in particular the variance σ_{bl} of a gaussian PSF from information contained in the image itself:

$$\text{PSF}(x,y) = \frac{1}{\sqrt{2\pi}\sigma_{bl}}\, e^{-(x^2+y^2)/(2\sigma_{bl}^2)}. \tag{2}$$

Our method can estimate the image blur σ_{bl} with an accuracy of about 10%. Other techniques for blur estimation using Gaussian PSF's [1,2] use derivatives of the Gaussian PSF to determine the variance of the Gaussian blur. We present an alternative method, which doesn't use derivatives, but a measure of the smoothness of the image at a certain position. This method can also be extended to gaussian PSF's that are not axially symmetrical and even to PSF's that aren't even gaussian. For out-of-focus blur, a uniform circular PSF is used [3,4]. Our method requires only minor modifications to adapt to this kind of PSF.

II. Our method

A. Principle

Our method for blur estimation is based on estimating the sharpness of the sharpest edges in the image. To analyse edges in the image, we calculate the Lipschitz exponent in all points where a change in intensity is found either in the horizontal or vertical direction. The Lipschitz exponent (sometimes referred to as Hölder exponent) is a measure of how smooth the image is in a certain point. In fact, it is an extension of how many times the image is differentiable in a certain point. For example, a signal that is differentiable once, has Lipschitz exponent 1, a step function has Lipschitz 0 and a dirac impulse Lipschitz -1. In the wavelet domain, it is possible to calculate the Lipschitz exponent in a certain point in the image from the evolution of the modulus maxima of the wavelet coefficients corresponding to that point through successive scales. Mallat has shown in [5–7] how Lipschitz regularity van be calculated for a one-dimensional signal.

Consider the cone of influence for a point v. The cone of influence in v (fig. 1) are the points (u,s) in scale-place space that are within the support of the wavelet $\psi_{v,s}$ at position v and scale s. Now, if the signal is uniformly Lipschitz α in the neighbourhood of a certain point v, then a constant A exists such that all wavelet coefficients within the cone of influence around v in the scale-place space satisfy the

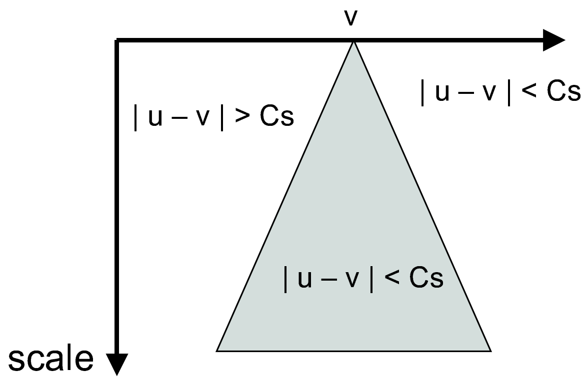


Fig. 1. Cone of influence for a point u.

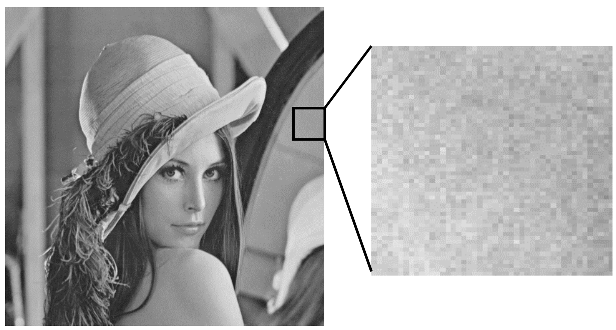


Fig. 2. Lena image and detail (mirror) which shows small intensity variations that disturb the blur estimation.

condition

$$\max(|Wf(u,s)|) = A\, s^{\alpha+1/2}, \tag{3}$$

which is equivalent to

$$\max(\log_2|Wf(u,s)|) = \log_2 A\ + \log_2(\alpha + 1/2). \tag{4}$$

Here, $|Wf(u,s)|$ represents the modulus of the wavelet transform of $f(x)$ at resolution scale s. The Lipschitz regularity in at v is given by the maximum slope of $\log_2|Wf(v,s,x)|$ as a function of $\log_2 s$ along the lines of modulus maxima that converge towards v within the cone of influence.

B. Practical considerations

The wavelet decomposition of the image is calculated, and by following the modulus maxima of the wavelet coefficients corresponding to a certain point in the image through different resolution scales, the Lipschitz exponent in that point is calculated by fitting an exponential curve to the modulus maxima versus the scale, as described earlier [5–7]. A problem in this approach is that even minor intensity variations in smooth regions result in Lipschitz exponents that correspond with sharp edges. An example is shown in figure 2. In the mirror region at the

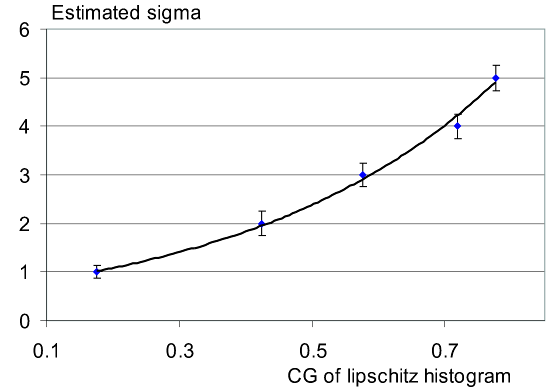


Fig. 3. The graph that shows the fitted relation between the estimated σ_{bl} and CG of the histogram of Lipschitz exponents.

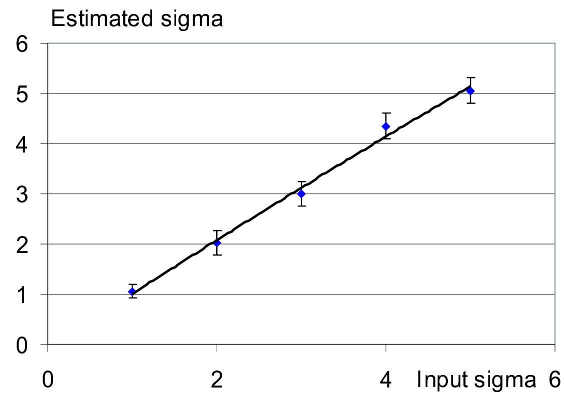
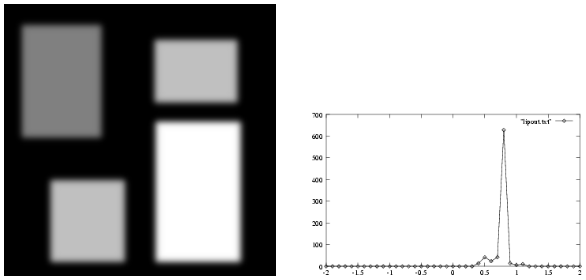


Fig. 4. Verification: the estimated σ_{bl} in function of the input σ_{bl} are on a perfect line .

right of the famous 'Lena' image, we can see what causes this effect. When magnified and with enhanced contrast, we see the intensity variations, even in apparently smooth regions. The problem is to distinguish sharp transitions whith a small amplitude from smooth transitions. This disturbs our estimation of the blurring of the image. However, transitions with small amplitude are not likely to belong to dominant image features. Because we work in the wavelet domain, we restrict our analysis to features that produce a gradient above a certain threshold. This gradient is extracted from the wavelet detail coefficients in the highest resolution scale. The threshold was determined empirically so that major image features were visible. Empirically, this thresholds corresponds with $30/\sigma_{bl}$.

From the Lipschitz exponents thus found along the significant edges in the image, a histogram is made. For this histogram, we divided the range of Lipschitz exponents in intervals with a width of 0.1. Because we restricted the lipschitz exponents to those corresponding with transitions with large amplitude, we already filtered out the sharpest transitions with a large amplitude in the image. When we



(a) synthetic image (b) Lipschitz histogram.

Fig. 5. Blur estimation on synthetic image.

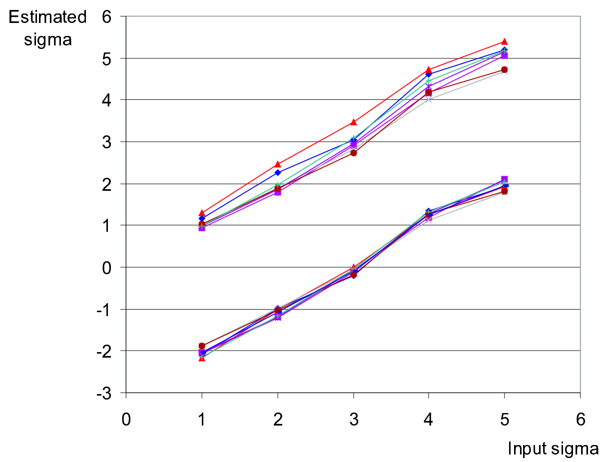


Fig. 6. Illustration of the offset effect when comparing the different images. The top set of curves is without subtracting the offset; the bottom set is after subtraction.

make a histogram of these Lipschitz exponents, we expect a single peak corresponding with the smoothness of the sharpest edges. When we have synthetic test images with large constant regions and step edges, we only have one kind of transitions, namely those step edges. When these edges are blurred, we obtain a histogram with one peak, corresponding with the sharpness of the blurred edges. This is illustrated in fig. 5. In reality, we have a certain distribution around this peak, from which we want to estimate the blur. Both the position of the maximum in the histogram and the center of gravity (CG) of the histogram are related to the blur in the image, but from experiments, the CG was the most reliable parameter. Let N_k be the number of transitions along significant edges in the image with Lipschitz exponent α_k, then CG is:

$$\mathrm{CG} = \frac{\sum_k N_k\, \alpha_k}{\sum_k N_k} \qquad (5)$$

C. Experiments

We studied a test set of eight images, taken from the Kodak website [8] and were taken with digital cameras. From these images, square regions were selected to reduce computation time. In each experiment, an image from this set was blurred with a gaussian PSF with σ_{bl} varying between 1 and 5. Each time, the Lipschitz exponents were calculated among the edges in the blurred image. For control purposes, they were plotted in a Lipschitz representation image, where an intensity is associated with the magnitude of the Lipschitz exponent. We made the histogram and calculated the CG the histogram.

An example of such an experiment for blurring with a Gaussian PSF with variance $\sigma_{bl} = 3$ is shown in figure 9. In fig. 9(a) the original image is shown, in fig. 9(b) we see the blurred image with $\sigma_{bl} = 3$. In fig. 9(c), a representation is made of which exponents contribute to the histogram in figure 9(d). In this representation, the Lipschitz exponent is plotted with black points corresponding to the sharpest transitions in the image; the smoother the transition, the lighter color was used. When fig. 9(c) is compared to fig. 9(a), one can verify that the considered Lipschitz exponents are indeed located among the sharpest edges with large amplitude in the image, though not all edges are found in 9(c) and not all dark points in 9(c) are edges. In our method, this is not a problem, since they are only used for gathering statistics.

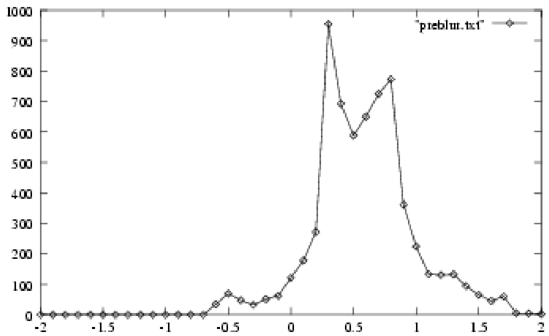
We calculated the Lipschitz exponent that corresponds to CG of the histogram, and determined the average $\mathrm{CG}_{\sigma_{bl}}$ over the whole set of test images blurred with the same σ_{bl}. To these data $(\sigma_{bl}, \mathrm{CG}_{\sigma_{bl}})$, an exponential curve was fitted (figure 3) experimentally, where the standard deviation over the experiments is shown as a vertical error bar. The fitting was

$$\sigma_{bl} = a\ \exp(b\ \mathrm{CG}_{\sigma_{bl}}) \qquad (6)$$

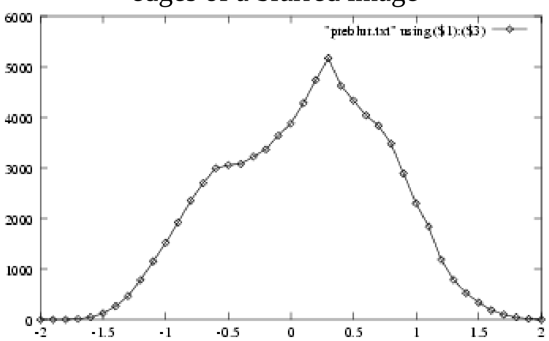
and for the parameters the fitting produced $a = 0.6645$ and $b = 2.6142$.

If we compare the estimated σ_{bl} to the input σ_{bl} with which the images were originally blurred, we obtain the graph in figure 4. In this graph, we can see that the estimations for σ_{bl} are accurate to about 10%.

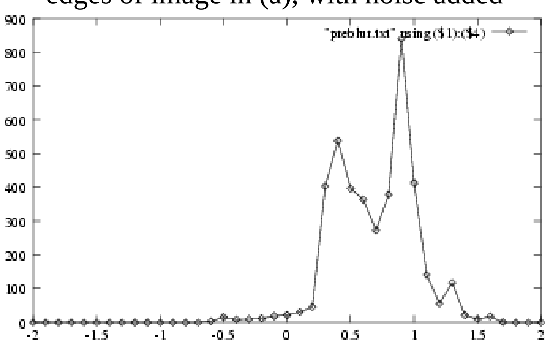
When we plot for all the images in all blurring experiments the estimated sigma versus the input sigma, we see that all the curves are more or less parallel. This suggests that in some images, there was already some initial blur (see top set of curves in figure 6). When this offset is subtracted from all curves, the standard deviation is a lot smaller (lower set of curves in figure 6). So what we estimate is the total effect of the blur that was already present in the original digital image, and the synthetic blur from the experiment.



(a) Histogram of lipschitz exponents along edges of a blurred image



(b) Histogram of lipschitz exponents along edges of image in (a), with noise added



(c) Histogram of lipschitz exponents along edges of image in (b) with extra smoothing applied as preprocessing

Fig. 7. Blur estimation used in restoration of a real image.

D. Robustness to noise

The technique described above is noise sensitive, as could have been expected for any technique based on finding local maxima. In applications like confocal microscopy or digital cameras, our initial experiments show that applying a median filter is a sufficient preprocessing step for reliable blur estimation because the noise is impulse like. In general cases however, when no precautions are taken, noise will disturb the blur estimation. There are two reasons for this. The first reason is because edges aren't detected accurately in the presence of noise. The second reason is that the lipschitz exponents on detected edges are disturbed.

In [9], the problem of edge detection in the presence of noise was handled by gaussian smoothing. In [1], this technique is incorporated in a probabilistic framework, and an expression is given how much an edge must be smoothed to obtain reliable detection. This degree of smoothing depends on the contrast of the edge, the original edge smoothness and the noise level. The minimal degree of extra smoothing required is called the minimum reliable scale for that position in the image. To actually compute this value, one needs the edge characteristics and the noise level, which are often unknown. In practice, the parameters are determined iteratively until a reliability criterium is satisfied. This is quite a time-consuming procedure.

We've performed some experiments with the technique described in [9] on our set of test images. We wanted to know how much smoothing was required in order to obtain a reliable blur estimation in the presence of noise. This additional smoothing was applied as a preprocessing step in our own blur estimation technique as described before. The additional blur value was later subtracted from the blur estimated in the smoothed image $\hat{\sigma}$ to obtain the original blur in the image:

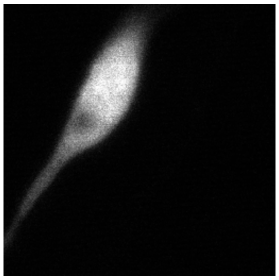
$$\sigma_{bl} = \sqrt{\hat{\sigma}^2 - \sigma_{postblur}^2} \qquad (7)$$

The effect of the additional blur is illustrated in figure 7. In figure 7 (a), the histogram of the lipschitz exponents along the edges a blurred image is shown. In (b), additive noise was added to the blurred image. This additive noise pollutes the histogram of the lipschitz exponents. In (c), we show the effect of applying gaussian smoothing to the image. Here, the shape of the original histogram of the lipschitz exponents returns, but it is shifted to larger vales because of the additional smoothing that we've applied (which is subtracted to obtain the final estimation).

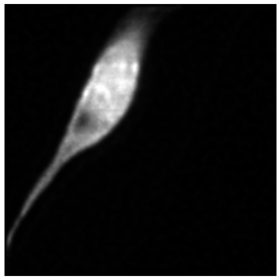
We can conclude from our experiments that in the majority of the cases (different blur and noise levels), the degree of smoothing required is $\sigma_{postblur} = 3$ pixels with an error margin of 1. This works when the blur is less than σ_{bl} = 6 and the noise less than σ_{noise} = 15. These are values encountered in realistic images.

E. Realistic applications of our method

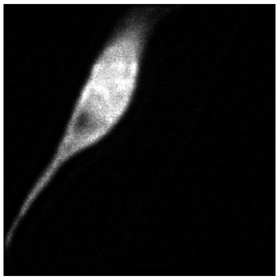
When we applied this method to estimate the blur in images for which no blur information was available, it was possible to use this estimation in a classical restoration scheme. In figure 8, a confocal microscope image of a cell nucleus of *Arabidopsis Thaliana* is shown. The left image shows the raw microscope image, the right image shows the image, restored with the well known Richardson-Lucy restoration algorithm [10], using the raw image and our



(a) Original microscope image.



(b) Image restored using our estimation.



(c) Image manually restored

Fig. 8. Blur estimation used in restoration of a real image.

estimate of the PSF as inputs. As a comparison, we also determined σ_{bl} manually by restoring the image with different values of σ_{bl} and selected the image with the best visual quality. The value of σ_{bl} that corresponds with this image, was the same as the one estimated with our method.

III. Conclusions and future work

In the experiments, we see that the CG of the histogram of Lipschitz exponents calculated among the edges in the image is a reliable parameter to estimate σ_{bl} of gaussian blur. However, the standard deviation on the estimate increases as σ_{bl} increases. Applying additional blur reduces the effect of noise in our blur estimation within an acceptable range of blur and noise values.

The tests were performed only for vertical edges in the image. Applying the algorithm to horizontal edges is similar, and will allow us to study Gaussian PSF's that are not circular symmetrical (with $\sigma_{bl,x} \neq \sigma_{bl,y}$).

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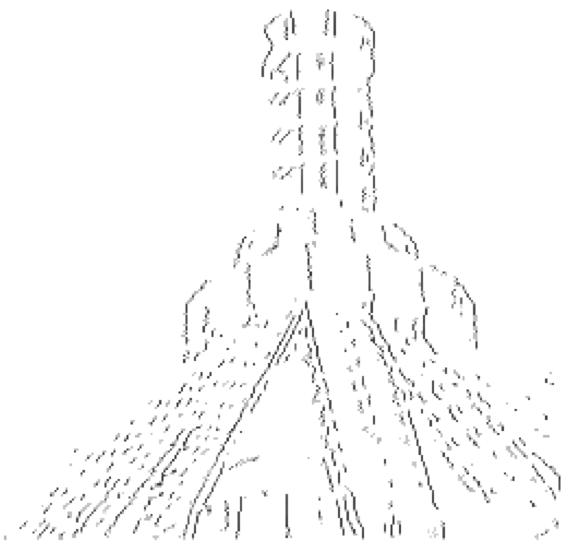
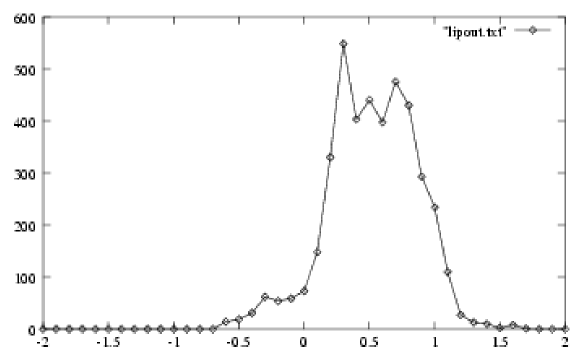
(a) Original image.

(b) Image blurred with $\sigma_{bl} = 3$.



(c) Lipschitz representation of the blurred image.



(d) Lipschitz histogram of the blurred image.

Fig. 9. Example of a blur estimation experiment on a test image.